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# A semidiscrete model for the scattering of light by vegetation

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**Abstract.** An advanced bidirectional reflectance factor model is developed to account for the architectural effects exhibited by homogeneous vegetation canopies for the first orders of light scattering. The characterization of the canopy allows the simulation of the relevant scattering processes as a function of the number, size, and orientation of the leaves, as well as the total height of the canopy. A turbid medium approach is used to represent the contribution to the total reflectance due to the light scattering at orders higher than 1. This model therefore incorporates two previously separate approaches to the problem of describing light scattering in plant canopies and enhances existing models relying on parameterized formulae to account for the hot spot effect in the extinction coefficient. Simulation results using this model compare quite favorably with those produced with a Monte Carlo ray-tracing model for a variety of vegetation cases. The semidiscrete model is also inverted against a well-documented data set of bidirectional reflectance factors taken over a soybean canopy. It is shown that the inversion of the model against a small subset of these measurements leads to reasonable values for the retrieved canopy parameters. These values are used in a direct mode to simulate the bidirectional reflectance factors for solar and viewing conditions significantly different from those available in the subset of soybean data and compared with the full set of actual measurements.

## 1. Introduction

Remote sensing technology is one way to document the properties of terrestrial surfaces and their actual and potential changes. Among other applications the knowledge of the state and evolution of the vegetation over these surfaces is of primary interest since plant canopies constitute the bulk of the biomass actively participating in the global carbon cycle and control the radiative boundaries in weather and climate models. The solar radiation incident on a plant canopy can be either absorbed, transmitted, or scattered in the medium, and the intensities of the relevant processes are determined by the physical properties of the system. Hence the radiance leaving the vegetation canopy, which may also be scattered and absorbed by the overlying atmosphere before being sensed by space instruments, is partly controlled by terrestrial surface properties. The goal of Earth observation remote sensing applications is to characterize the properties of the geophysical media which have significantly affected the measured radiation. This is a typical inverse problem, where the variables of a radiation transfer model are adjusted to best explain the observed variability in the measured reflectances.

Most physically based models of radiation transfer in geophysical media such as vegetation and soil are derived from the classical theory developed by *Chandrasekhar* [1960]. It is assumed that the canopy is composed of infinitely small leaves oriented in arbitrary directions [*Pinty and Verstraete*, 1997]. A number of mathematical expressions have been developed in the past [*Bunnik*, 1978; *Ross*, 1981; *Goel and Strebel*, 1984; *Kimes*, 1984; *Verstraete*, 1987] to represent the distribution of leaf normal orientation (i.e., planophile, erectophile, uniform, etc.) which controls the extinction and scattering efficiencies of a plant canopy medium. The extension of the standard radiative transfer equation to the plant canopy problem in the optical spectral domain requires additional modifications of the coefficients representing these extinction and scattering efficiencies. These modifications aim at describing specific effects due to the finite size of the leaves as suggested in previous studies by, for instance, *Ross* [1981], *Marshak* [1989], *Myneni et al.* [1989], *Knyazikhin et al.* [1992], *Iaquinta and Pinty* [1994], and *Liang and Strahler* [1993]. Indeed, it has been demonstrated that the finite size of the leaves leads to a specific effect known as the hot spot phenomenon corresponding to an increase in the reflectance field particularly noticeable for observation directions close to the illumination direction [e.g., *Nilson and Kuusk*, 1989; *Verstraete et al.*, 1990; *Jupp and Strahler*, 1991]. The proposed modification of the extinction coefficient is achieved by averaging the properties of the vegetation medium, such as a length correlation related to the scattering events or the average radius of typical Sun flecks on the leaves. In practice, this information is mainly accessible by solving the inverse problem against field data. *Marshak* [1989], *Knyazikhin et al.* [1992], *Iaquinta and Pinty* [1994], and *Iaquinta* [1995] proposed the required extension of the original radiation transfer equation for the hot spot effect in the first orders of scattering. As discussed by *Myneni et al.* [1991a], the radiation transfer problem becomes practically unsolvable when this latter effect is added in the multiple-scattering regime.

The intrinsic limitations of the extended turbid medium concept is discussed at length by *Pinty and Verstraete* [1997]. In

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addition to the pseudoturbid representation of a plant canopy, the radiation transfer equation assumes a continuous medium since the derivatives of the intensities along the three Cartesian coordinates are always defined. However, this may not always be justified, especially when the vegetation canopy is constituted by a limited number of large leaves since the medium then strongly departs from a continuous turbid medium. Clearly, these structural and geometrical features affect the value of the optical thickness of the vegetation canopies, and as a result, the variables describing the architecture of the system become state variables of the radiative problem. Basically, the complete description of this geophysical medium necessitates an explicit representation of each leaf size and location in three-dimensional space, even in the case of a horizontally homogeneous canopy. Solving such a radiative transfer problem requires the use of computationally expensive Monte Carlo ray-tracing methods [e.g., *Ross and Marshak*, 1988; *Govaerts*, 1996] which may be inappropriate or too difficult to handle for global applications.

The objective of this study is to develop a generic radiation transfer model which uses a statistical description of the spatial distribution and geometrical properties of the leaves (e.g., size and orientation) to represent the vegetation canopy. Typically, the average size of the leaves, their number per unit volume, and the total height of the canopy constitute a set of independent architectural parameters from which the leaf area index, a biophysical variable of great interest in various applications, can be straightforwardly computed. The solution of the radiation transfer problem using such a limited but statistical representation of the vegetation canopies can be formally expressed at all orders of scattering following the spatial discretization of the medium. It is, however, noticeable that the impact of the discrete properties of a canopy as compared to a turbid-like medium is mainly significant for the very first few orders of scattering. Indeed, it may reasonably be expected that the multiple-scattering events tend to smear out the specific architectural features of the vegetation canopy. This has long been recognized by *Marshak* [1989] and others (including, for instance, *Liang and Strahler* [1993] and *Iaquinta and Pinty* [1994]), who recommended finding separate analytical solutions for the intensities that have been scattered only once, either by the underlying soil or by the leaves. The remaining contributions due to higher orders of scattering can be estimated by more classical radiation transfer methods applicable to the case of turbid media. We adopted a similar approach in finding the full solution of the radiation transfer problem for our semidiscrete model: The first orders of scattering (by the soil and by the leaves) are calculated in three-dimensional space after an adaptation of the original discrete model developed by *Verstraete* [1987] for the extinction of the direct incoming solar radiation. Both contributions depend on the absolute direction of the incoming solar radiation defined by the solid angle $\Omega_0$ and the position of the sensor defined by the solid angle $\Omega$. The statistical representation of the canopy architecture allows us to model explicitly the hot spot phenomenon in the first two orders of scattering following an adaptation of the model developed by *Verstraete et al.* [1990]. Although other representations of the hot spot have been published in the literature [e.g., *Jupp and Strahler*, 1991; *Kuusk*, 1991], the formulation chosen here logically fits into the discrete approach we use for describing the state of the canopy. The multiple-scattering contribution is calculated with a discrete ordinates method using an azimuthally averaged expression of the anisotropic scattering phase function proposed by *Shultis and Myneni* [1988].

This model thus constitutes the latest advance in representing the transfer of radiation in a homogeneous plant canopy, capitalizing on the progress achieved over the last 10 years, in particular the discrete representation of a plant canopy (as opposed to a purely turbid medium) for the first orders of scattering as well as an accurate description of the multiple-scattering processes for an optically finite medium. This approach is supported by the accuracy with which this model can simulate the reflectance field on the basis of measurable canopy properties, by the speed of execution of the model, and by its ability to represent the main physical processes involved.

## 2. Specification of the Radiative Properties of a Discrete Canopy

The vegetation canopy, which is assumed to be composed only of leaves, can be stratified into $k = 1, 2, \cdots, K$ layers to describe the vertical structure of the medium in a discrete way. Each layer $k$, located between the two levels $z_{k-1}$ and $z_k$ (level $z_0$ is at the top of the canopy and level $z_K$ at the bottom) contains $n_k^l$ leaves per unit area, where the index $l$, $l = 1, 2, \cdots, L$ denotes the various types of leaves in the layer $k$. These idealized leaves are assumed to be flat and of area $a_k^l$ (square meters).

In each layer $k$ the elementary leaf area index $\lambda_k^l$ corresponding to each type of leaf is computed as the product of the number $n_k^l$ of leaves of that type in that layer by the one-sided area $a_k^l$ of these leaves, so that the leaf area index $\lambda_k$ of that layer and for all types of leaves is

$$\lambda_k = \sum_{l=1}^{L} n_k^l a_k^l \tag{1}$$

The cumulative leaf area index $\mathrm{LAI}_k$ from the top of the canopy down to level $z_k$ is the sum of all contributions $\lambda_k$ from these layers:

$$\mathrm{LAI}_k = \sum_{i=1}^{k} \lambda_i = \sum_{i=1}^{k} \sum_{l=1}^{L} n_i^l a_i^l \tag{2}$$

and the total leaf area index LAI of the canopy is defined as the cumulative one-sided area of all leaves per square meter:

$$\mathrm{LAI} = \sum_{k=1}^{K} \lambda_k = \sum_{k=1}^{K} \sum_{l=1}^{L} n_k^l a_k^l = \mathrm{LAI}_K \tag{3}$$

If the canopy is sufficiently homogeneous to be represented by average statistical properties, the mean leaf area ($a_\ell$, in square meters) and the mean leaf number density ($n_v$, $\mathrm{m}^{-3}$) can be defined by

$$a_\ell = \sum_{k=1}^{K} \sum_{l=1}^{L} n_k^l a_k^l \Bigg/ \sum_{k=1}^{K} \sum_{l=1}^{L} n_k^l \tag{4}$$

$$n_v = \sum_{k=1}^{K} \sum_{l=1}^{L} n_k^l \Bigg/ H \tag{5}$$

where $H$ is the height of canopy (in meters), so that the total canopy leaf area index can also be estimated as

$$\mathrm{LAI} = n_v a_\ell H = \Lambda H \tag{6}$$

where $\Lambda$ is the average leaf area density of the canopy ($n_v\, a_\ell$ in square meters per cubic meters). In practice, canopies with leaves of different types and dimensions will be simulated as being composed of equivalent leaves with the statistical properties defined above.

Figure 1 illustrates the variation of the leaf area density $\Lambda$ due to changes in the values of the two parameters $n_v$ and $d_\ell$ used to specify the architecture of the homogeneous canopy, where $d_\ell$ is the equivalent diameter corresponding to $a_\ell$. The application of the turbid medium concept which requires numerous and infinitely small but widely dispersed scattering elements is mainly limited to the architectural conditions prevailing on the left-hand side of the figure. Conversely, the larger the leaves and the smaller the number of scattering elements, the more a model capable of handling finite canopy architectures is needed.

The vegetation canopy is thus idealized as a large but finite number of flat leaves, uniformly distributed in space between the underlying soil and the level of the top of the canopy. The canopy is characterized by a suitable leaf orientation distribution. The vegetation canopy simulated here thus differs from a classical turbid medium because it explicitly recognizes the presence of finite scatterers. This simple fact implies that the radiation interception process by the leaves will not follow a continuous scheme, and in particular that radiation will be allowed to travel unimpeded within the free space between the scatterers. Since extinction occurs only at discrete locations, some of the radiation emerging from an arbitrary source, located inside or outside the canopy, may occasionally travel without any interaction in this canopy, depending on the shape and size of free space between these leaves.

In general, when the radiation is scattered back in the direction from which it originates, the probability of further interaction with the canopy is lower than in other scattering directions. In the case of remote sensing measurements outside the canopy, this gives rise to a relative increase in the bidirectional reflectance factor in the backscattering direction which is known as the hot spot phenomenon. Of course, this effect is not limited to the incoming light of the direct solar beam: Any and all radiation traveling inside a plant canopy will propagate according to the probability of occurrence of volumes free of scatterers and of the presence of leaves. The importance of the hot spot effect as an element of the bidirectional reflectance of plant canopies results from the fact that direct solar radiation constitutes the main source of light in the case of remote sensing.

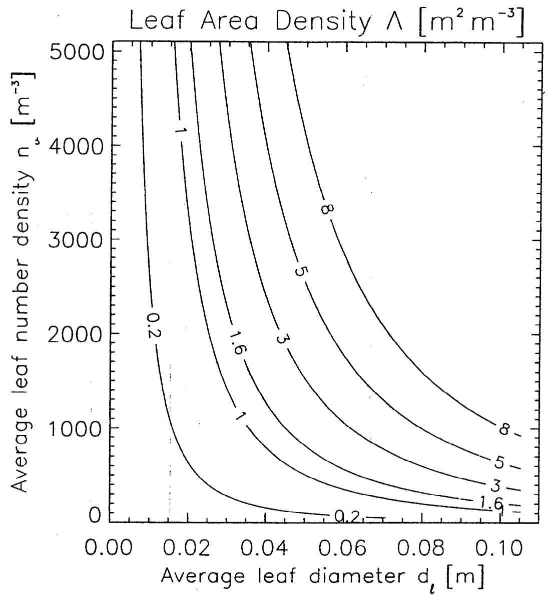


**Figure 1.** Graph of the leaf area density of a canopy as a function of the average leaf diameter and the average leaf number density.

## 2.1. Representation of the Extinction Process for Direct Incoming Solar Radiation

For the purpose of the following discussion, it is necessary to settle on a particular geometrical frame of reference. Specifically, the $z$ axis will be directed downward along the local vertical, and the direction of the radiation's path will be specified by a vector defined by an azimuth angle $\phi$ with respect to an arbitrary but fixed absolute direction and a zenith angle $\theta$ with respect to the upward vertical. The radiation $I_\downarrow$, in particular the one impinging at the top of the canopy, is then oriented downward, with $\mu = \cos\theta < 0$, and the radiation traveling in the upward direction is noted $I_\uparrow$ with $\mu > 0$.

The extinction process in a vegetation layer is crucially dependent on the orientation, density, and number of the structural canopy elements capable of intercepting radiation traveling along a particular direction, as discussed by *Nilson* [1971]. In the present paper we follow the approach proposed by *Verstraete* [1987], where the probability for the radiation to be transmitted through an elementary layer containing $n_\ell$ leaves of average size $a_\ell$ per unit area is given by

$$0 \le P(\Omega_0) = 1 - a_\ell n_\ell \frac{G(\Omega_0)}{|\mu_0|} \le 1 \tag{7}$$

In this equation, the values $a_\ell$, $G(\Omega_0)$, and $\mu_0$ are imposed by the system under study, while $n_\ell$ is the only parameter that can be modified to ensure that this probability remains within the stated limits. Since all variables in the right-hand side of the equation are positive, $P(\Omega_0)$ will never exceed 1. On the other hand, the number of model layers is selectable and must be set large enough to avoid self-shading within any given layer. In practice, this minimal number of layers will depend on the size and orientation of individual leaves, on the LAI of the canopy, and on the particular geometry of illumination and observation. Given that each layer must contain at least one leaf, higher numbers of layers may result in more accurate results, but also in higher computing costs. For the purpose of this paper, we have selected $n_\ell$ so that the leaf area index of each layer is numerically equal to 1% of $|\mu_0|\mu/G(\Omega_0)G(\Omega)$.

This equation implies that the discretization of the canopy in the vertical direction is made in such a way that the leaves do not overlap within any one of the layers, and that their orientation follows the well-known $G(\Omega)$ functions defined by *Ross* [1981] to represent the mean projection of a unit foliage area in the direction $\Omega$ for which the cosine zenith angle is $\mu$, that is,

$$G(\Omega) = \frac{1}{2\pi} \int_{2\pi} g(\Omega_\ell)|\Omega_\ell \cdot \Omega|\, d\Omega_\ell \tag{8}$$

where $g(\Omega_\ell)$ is the probability density function for the leaf normal distribution ($\Omega_\ell$) which can be specified using the trigonometrical formulae suggested by *Bunnik* [1978], for instance.

When the vegetation medium is divided into $K$ layers, assumed to be radiatively independent of each other in the sense defined above, the probability of transmitting radiation down to an arbitrary level $z_k > 0$ is the product of the transmission probabilities assigned to each of the elementary layers between the level $z_0$ at the top of the canopy and the level $z_k$:

$$P(z_k, \Omega_0) = \prod_{i=1}^{k} \left[ 1 - a_\ell n_\ell \frac{G(\Omega_0)}{|\mu_0|} \right] \tag{9}$$

Furthermore, since the products $a_\ell n_\ell$ are assumed to be identical for all $k$ layers in the simulated canopy, the layer leaf area index $\lambda_k$ will be denoted $\lambda$ and

$$P(z_k, \Omega_0) = \left[ 1 - \lambda \frac{G(\Omega_0)}{|\mu_0|} \right]^k \tag{10}$$

When the number of layers $K$ tends to infinity and $a_\ell$ tends to zero, i.e., when the canopy is made up of a large number of small-sized leaves, then the discrete equation (10) reduces to the classical continuous extinction law for a turbid medium:

$$P(z_k, \Omega_0) = \exp \left[ - \frac{G(\Omega_0)}{|\mu_0|} \mathrm{LAI}_k \right] \tag{11}$$

This approach and similar equations have been widely published in the literature over the last 40 years; see, among others, *Monsi and Saeki* [1953], *Norman* [1975], *Ross* [1981], and *Verstraete* [1987].

Any difference between the estimations provided by (10) and (11) may be used to quantify the degree of approximation and the accuracy with which a plant canopy can in fact be considered a turbid medium, as opposed to a set of discrete leaves of finite size, as far as the penetration of incoming direct solar radiation is concerned.

### 2.2. Representation of the Extinction Process for Singly Scattered Outgoing Solar Radiation

As discussed before, the extinction process for the singly scattered solar radiation traveling upward is also controlled by the existence of free spaces between leaves inside the canopy. Following the original development proposed by *Verstraete et al.* [1990] and *Pinty and Verstraete* [1997] it can be demonstrated that the actual optical thickness for the radiation scattered in the direction $\Omega$ can be expressed by

$$\tau(z, \Omega, \Omega_0) = \frac{\|\mathcal{V}_\bullet\|}{\|\mathcal{V}_2\|} \int_z^0 \tilde{\sigma}(z', \Omega)\, dz' \tag{12}$$

where $\tilde{\sigma}(z, \Omega) = G(\Omega)\Lambda(z)$ is the purely turbid extinction scattering coefficient taken at level $z$ and along the direction $\Omega$. The correcting factor $\|\mathcal{V}_\bullet\|/\|\mathcal{V}_2\|$ is given by

$$1 - \frac{2}{\pi \zeta_T} [\zeta_* \cos^{-1} \zeta_* - (1 - \zeta_*^2)^{1/2} + \tfrac{1}{3} \sin^3 (\cos^{-1} \zeta_*) + \tfrac{2}{3}] \tag{13}$$

where $\zeta_* = \min(1, \zeta_T)$ with $\zeta_T = zG_f/2r$, and $G_f$ is a geometric factor defined by

$$G_f = [\tan^2 \theta_0 + \tan^2 \theta - 2 \tan \theta_0 \tan \theta \cos (\phi_0 - \phi)]^{1/2} \tag{14}$$

In their development, these authors idealized the space free of scatterers with respect to the incoming solar radiation by considering two basic cylindrical volumes $\|\mathcal{V}_1\|$ and $\|\mathcal{V}_2\|$ drawn along the incoming $\Omega_0(\theta_0, \phi_0)$ and outgoing $\Omega(\theta, \phi)$ directions. $\|\mathcal{V}_1\|$ and $\|\mathcal{V}_2\|$ have a common base defined by a circular Sun fleck of radius $r$ and therefore share a common volume free of scatterers. The volume $\|\mathcal{V}_\bullet\|$ represents the complementary space which is not commonly shared by two basic cylindrical volumes $\|\mathcal{V}_1\|$ and $\|\mathcal{V}_2\|$ [see *Verstraete et al.*, 1990, Figure 1].

*Verstraete et al.* [1990] suggested that the mean Sun fleck radius $r$ may be approximated by the following expression:

$$r = r_0 \sqrt{\frac{|\mu_0|}{G(\Omega_0)\Lambda z_p}} \tag{15}$$

where $r_0$ is the average radius of the "hole" between the leaves in the top layer of canopy, and $z_p$ the distance of penetration of the direct solar radiation. Formally, and somewhat arbitrarily, the total area of the top canopy layer can be decomposed into a set of nonoverlapping leaves projected on a horizontal plane, and an associated set of holes satisfying the following equation:

$$A = n_0 a_h + n_0 a_\ell \frac{G(\Omega_0)}{|\mu_0|} \tag{16}$$

where $n_0$ is the number of leaves in this top layer, and where this equation defines $a_h$, the nominal area of a single hole at the top of canopy. This formalism, which assumes an equal number of holes and of leaves, implies an isotropic distribution of holes described by the same statistical laws as the distribution of leaves. This apparent average area of an elementary hole can thus be estimated as

$$a_h = \frac{1}{n_0} \left[ 1 - \lambda_0 \frac{G(\Omega_0)}{|\mu_0|} \right] \tag{17}$$

where this equation applies to a layer at the top of the canopy whose thickness implies a predefined level of absorption. The leaf area index of and the number of leaves in that layer are then denoted $\lambda_0$ and $n_0$, respectively. Combining (16) and (17) leads to the following expression for $r_0$:

$$r_0 = \sqrt{\frac{1}{\pi n_0} \left[ 1 - \lambda_0 \frac{G(\Omega_0)}{|\mu_0|} \right]} \tag{18}$$

According to (15), the Sun fleck radius $r$ therefore depends on the solar zenith angle, the leaf angle distribution, and the density and size of the leaves. Comparisons made with a Monte Carlo ray-tracing model to be discussed in section 4.1 indicate that for all practical purposes the relative amounts of absorption corresponding to the distance of penetration $z_p$ and the top layer $\lambda_0$ can be fixed at 99.5% and 10%, respectively.

The conceptual approach described above is implemented in the discrete formulation as a modification of the optical thickness of the canopy. Following (12), the optical thickness at a given level $z_k$ is expressed by

$$\tau(z_k, \Omega, \Omega_0) = \frac{\|\mathcal{V}_\bullet\|}{\|\mathcal{V}_2\|} G(\Omega)\, \mathrm{LAI}_k \tag{19}$$

where $\mathrm{LAI}_k = k\lambda$, and where the correcting factor $\|\mathcal{V}_\bullet\|/\|\mathcal{V}_2\|$ is evaluated at level $z_k$. An equivalent elementary leaf area index $\lambda'_k$ is then defined for each elementary layer located between the levels $z_k$ and $z_0$ by

$$\lambda'_k = \frac{\|\mathcal{V}_\bullet\|}{\|\mathcal{V}_2\|}\,\lambda \tag{20}$$

where the dependency of $\lambda'_k$ on $\Omega$ and $\Omega_0$ is implicit. According to (19) and (20), the elementary leaf area index can be calculated for any illumination and observation conditions given the knowledge of the statistical architecture of the canopy, namely the orientation, size, and density of leaves. The probability of transmitting radiation through the actual path between the bottom of the canopy $z_K$ and the level $z_k$ is then

$$P(z_k, \Omega, \Omega_0) = \prod_{j=K}^{k+1} \left[1 - \lambda'_K \frac{G(\Omega)}{\mu}\right] \tag{21}$$

Clearly, when the size of the leaves goes to zero, (21) reduces to the functional form of (10) since we converge toward the solution appropriate for a turbid scattering medium.

### 2.3. Representation of the Scattering Properties of the Leaves

Once the radiation is intercepted by the vegetation elements, it is scattered in directions determined by the leaf orientation and the leaf phase function. The combination of these two effects is needed to represent the leaf scattering function which, unlike most atmospheric particles, does not exhibit rotational invariance. We adopt a classical plate scattering model for the leaves which requires the specification of the relative fractions $r_\ell$ and $t_\ell$ of the intercepted energy which are reflected and transmitted following a simple cosine distribution law around the normal to the leaves. The leaf scattering function is then written as [e.g., *Shultis and Myneni*, 1988; *Knyazikhin et al.*, 1992]

$$f(\Omega' \to \Omega, \Omega_\ell) = \frac{r_\ell |\Omega \cdot \Omega_\ell|}{\pi} \qquad (\Omega \cdot \Omega_\ell)(\Omega' \cdot \Omega_\ell) < 0 \tag{22}$$

$$f(\Omega' \to \Omega, \Omega_\ell) = \frac{t_\ell |\Omega \cdot \Omega_\ell|}{\pi} \qquad (\Omega \cdot \Omega_\ell)(\Omega' \cdot \Omega_\ell) > 0 \tag{23}$$

This formulation could be extended to account for more radiatively complex processes which may exist at the scale of the leaf, such as the specularity phenomenon [e.g., *Vanderbilt*, 1985; *Marshak*, 1989; *Govaerts*, 1996]. As far as the single scattering is concerned, it would be sufficient to modify the phase function to describe this phenomenon. To take specularity effects into account in the multiple scattering, however, the model would also have to keep track of the angular distribution of the illumination of each leaf, and this would result in increased demands in computer resources. This approach was not followed here because it would have resulted in the use of incompatible phase functions in various parts of the model.

### 2.4. Representation of the Boundary Conditions

The formulation of the radiative transfer problem requires the specification of the external sources of radiation at the top and bottom of the vegetation layer.

The upper boundary receives multiple sources of radiation which can be arbitrarily decomposed into a direct and a scattered component. A correct representation of this boundary condition would therefore require solving the full coupled problem including soil, vegetation, and atmospheric layers. Here again, the cost of the coupling is prohibitive, and in the frame of our model development for terrestrial surfaces, we will assume that the precise radiative coupling between the geophysical layers embracing the vegetation can be done at a later stage. Accordingly, and unless stated otherwise, we will consider only the direct component of the solar flux to represent straightforwardly the upper boundary condition.

The second boundary condition depends on the radiative properties of the underlying soil which may scatter radiation in three-dimensional space in a more or less complex form. It basically depends on the distribution of incoming intensities that have traveled and have been scattered by leaves through the vegetation layer, and the bidirectional reflectance factor of the soil itself. Clearly, a variety of reasonable assumptions can be made at this lower boundary condition. In our model, we will consider that the direct radiation which is reaching the soil underneath without being intercepted by the leaves can be scattered by the soil following either a Lambertian soil property or a bidirectional reflectance factor. When the former condition applies, the required variable is then the soil bihemispherical reflectance factor or albedo $R_s$ and, in the latter case, we will use the adaptation of the original Hapke's model [*Hapke*, 1981] as modified by *Pinty et al.* [1989].

## 3. Derivation and Solution to the Radiative Transfer Equations

*Marshak* [1989] demonstrated the advantages of deriving and solving separately the equations for the radiation that has interacted once with the soil only (the so-called uncollided radiation), that which has interacted once with the leaves only (first collided) and the remaining radiation (multiply collided) which has been scattered more than once by the soil-vegetation system. Indeed, equations for uncollided and first collided intensities can be explicitly written and solved exactly, in particular to permit the inclusion of a formulation of the hot spot effect as a function of the architectural parameters, whereas the multiple scattering intensities require a numerical solution as explained below. Since the radiation can be scattered in many different directions, the derivation of the radiation transfer equation for the multiply scattered intensities requires one to account for the presence of volumes free of scatterers inside the vegetation canopy. However, as mentioned earlier, the scattering events tend to smear out the effects due to the finite size of the leaves, partly because typical leaf phase functions do not exhibit very sharp angular behavior. We will then solve the multiple-scattering contribution using a standard turbid medium theory (i.e., the leaves behave as "oriented point" scatterers).

### 3.1. Intensity Scattered Once by the Soil Only (Uncollided Intensity)

Given a collimated radiation beam $I^0(z_0, \Omega_0)$ at the top of the canopy (as provided by direct solar radiation), the downward uncollided intensity $I^0_\downarrow(z_k, \Omega, \Omega_0)$ at level $z_k$ below the top of the canopy in the same direction is derived from (10):

$$I^0_\downarrow(z_k, \Omega_0) = I_s\left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^k \tag{24}$$

where $I_s = I^0(z_0, \Omega_0)\delta(\Omega - \Omega_0)$ is the incoming collimated beam at the top of the canopy and where the bracketed factor represents the probability that a light ray within this beam is transmitted downward in the same direction through the first $k$ layers, or, alternatively, the fraction of the beam transmitted downward without interception. The collimated radiance received by the soil underlying the canopy is obtained by replacing $z_k$ by $H$ and $k$ by $K$.

The radiation scattered by the soil, which provides the lower radiative boundary condition of the canopy system for uncollided radiation, is given by

$$I^0_{\uparrow}(H, \Omega, \Omega_0) = \frac{1}{\pi} \gamma_s(H, \Omega, \Omega_0)|\mu_0|I_s\left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^K \quad (25)$$

where $\gamma_s(H, \Omega, \Omega_0)$ denotes the bidirectional reflectance factor of the soil at the bottom of the canopy. Within the canopy, the radiance $I^0_{\uparrow}(z_k, \Omega, \Omega_0)$ which has been scattered by the soil only but not by the canopy, the so-called uncollided intensity, in the upward direction $\Omega$ and at level $z_k$, is given by the product of the radiation scattered by the soil $I^0_{\uparrow}(H, \Omega, \Omega_0)$ and the probability of transmitting radiation through the actual path between the bottom of the canopy and level $z_k$ (i.e., equation (21)):

$$I^0_{\uparrow}(z_k, \Omega, \Omega_0) = \frac{1}{\pi} \gamma_s(H, \Omega, \Omega_0)|\mu_0|I_s \cdot \left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^K \prod_{j=K}^{k+1} \left[1 - \lambda'_K \frac{G(\Omega)}{\mu}\right] \quad (26)$$

Applying (26) at the top of the canopy (level $z_k = z_0$) and normalizing by the intensity of the source of incoming direct solar radiation ($I_s$) as well as by the reflectance of a Lambertian surface illuminated and observed under identical conditions ($|\mu_0|/\pi$) yields the contribution to the bidirectional reflectance factor due to this uncollided radiation, namely,

$$\rho^0(z_0, \Omega, \Omega_0) = \gamma_s(H, \Omega, \Omega_0) \cdot \left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^K \left[1 - \lambda \frac{\|\mathcal{V}_\bullet\|}{\|\mathcal{V}_2\|} \frac{G(\Omega)}{\mu}\right]^K \quad (27)$$

### 3.2. Intensity Scattered Once by the Leaves Only (First Collided Intensity)

The first collided intensity in downward direction $\Omega$ corresponds to the radiation scattered only once by leaves, but which has not interacted with the soil. This intensity is calculated from the downward source of radiation $Q^0_{\downarrow}(z_k, \Omega, \Omega_0)$ coming from the direct intensity, attenuated through the first $k$ layers, and scattered by the leaves according to their optical properties in layer $k$:

$$Q^0_{\downarrow}(z_k, \Omega, \Omega_0) = \frac{1}{\pi} \Gamma(z_k, \Omega_0 \to \Omega) I_s\left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^k \quad (28)$$

where $\Gamma(z_k, \Omega_0 \to \Omega)$ is the canopy scattering phase function for direct radiation within layer $k$. Since the leaf properties are assumed to be the same throughout the canopy, we have [*Shultis and Myneni*, 1988; *Knyazikhin et al.*, 1992]

$$\frac{1}{\pi}\Gamma(z_k, \Omega' \to \Omega) \equiv \frac{1}{\pi}\Gamma(\Omega' \to \Omega) \equiv \frac{1}{2\pi}\int_{2\pi} g(\Omega_\ell)|\Omega' \cdot \Omega_\ell| f(\Omega' \to \Omega, \Omega_\ell)\, d\Omega_\ell \quad (29)$$

The downward intensity for the first order of scattering, at an arbitrary level $z_k$ of the canopy, is the sum of all internal sources from the different levels $i = 1, 2, \cdots, k$, attenuated along the distance between the levels $z_i$ and $z_k$. Consequently, the expression giving the downward intensity is

$$I^1_{\downarrow}(z_k, \Omega, \Omega_0) = \frac{1}{|\mu|} \sum_{i=1}^{k} Q^0_{\downarrow}(z_i, \Omega, \Omega_0)\lambda\left[1 - \lambda \frac{G(\Omega)}{|\mu|}\right]^{k-i} \qquad k > 0 \quad (30)$$

Similarly, the corresponding expression for the upward intensity is

$$I^1_{\uparrow}(z_k, \Omega, \Omega_0) = \frac{1}{\mu} \sum_{i=K}^{k+1} \left\{ Q^0_{\downarrow}(z_i, \Omega, \Omega_0)\lambda \prod_{j=i}^{k+1} \left[1 - \lambda'_i \frac{G(\Omega)}{\mu}\right]\right\} \quad (31)$$

$$k < K$$

The contribution to the bidirectional reflectance factor due to the first collided intensity is obtained after normalizing the upward intensity $I^1_{\uparrow}(z_0, \Omega, \Omega_0)$ by the incoming directional source of radiation at the top of the canopy:

$$\rho^1(z_0, \Omega, \Omega_0) = \frac{\Gamma(\Omega_0 \to \Omega)}{\mu|\mu_0|} \sum_{i=K}^{1} \lambda\left[1 - \lambda \frac{G(\Omega_0)}{|\mu_0|}\right]^i \cdot \left[1 - \lambda \frac{\|\mathcal{V}_\bullet\|}{\|\mathcal{V}_2\|} \frac{G(\Omega)}{\mu}\right]^i \quad (32)$$

### 3.3. Multiply Scattered Intensity

As mentioned in the introduction, the multiple-scattering contribution to the total reflectance is approximated by the solution of a radiation transport equation applied to the case of infinitely small but oriented scatterers (turbid medium solution):

$$-\mu \frac{\partial I(z, \Omega)}{\partial z} + \tilde{\sigma}(z, \Omega)I(z, \Omega) = \int_{4\pi} \tilde{\sigma}_s(z, \Omega' \to \Omega)I(z, \Omega')\, d\Omega' \quad (33)$$

where $\tilde{\sigma}$ ($m^{-1}$) and $\tilde{\sigma}_s$ ($m^{-1}$ $sr^{-1}$) are the extinction and differential scattering coefficients taken at point $z$ and along the direction $\Omega$, respectively. The discrete ordinates method provides an accurate solution to (33) [e.g., *Myneni et al.*, 1991b]. However, since this technique requires the discretization of the angular domain into specific quadrature angles, it becomes computationally expensive if a large number of angles are needed to describe accurately the scattered field. In practice, the canopy scattering phase function $\Gamma$ is smooth enough in the azimuthal plane to justify the use of an azimuthally averaged radiation transfer equation for the plant canopy problem. Therefore, (33) reduces to

$$-\mu \frac{\partial I(z, \mu)}{\partial z} + G(z, \mu)I(z, \mu) = 2\int_{-1}^{1} \Gamma_M(\mu' \to \mu)I(z, \mu')\, d\mu' \quad (34)$$

where $\Gamma_M(\mu' \rightarrow \mu)$ represents the "one-angle" scattering phase function under the assumptions of a leaf angle distribution independent of azimuth and a bi-Lambertian scattering phase function. *Shultis and Myneni* [1988, equations (44)–(47)] proposed the formulation used here.

The lower boundary condition applicable to this multiple-scattering problem is given by the azimuthal average of the bidirectional reflectance factor emerging from the soil. In order to ensure internal consistency, the bidirectional reflectance factor of the soil is computed using the soil model used earlier to scatter uncollided radiation.

The bidirectional reflectance factor $\rho^M(z_0, \mu_0, \mu)$ is derived after normalizing the multiply scattered intensity emerging at the top of canopy by the azimuthal average of the source, as was done earlier (see (27)), namely,

$$\rho^M(z_0, \mu_0, \mu) = \frac{I^M(z_0, \mu_0, \mu)}{(|\mu_0|/\pi)\int_0^{2\pi} I_s\, d\phi} = \frac{I^M(z_0, \mu_0, \mu)}{2I_s|\mu_0|} \tag{35}$$

Figures 2 and 3 illustrate the differences computed in the principal plane between the "one-angle" and the "exact" solutions for the multiple-scattering contribution. These differences have been normalized by the averaged reflectance and expressed in percents:

$$\varepsilon = 200\,\frac{\rho^M(z_0, \mu_0, \mu) - \rho^M(z_0, \Omega_0, \Omega)}{\rho^M(z_0, \mu_0, \mu) + \rho^M(z_0, \Omega_0, \Omega)} \tag{36}$$

The reference (so-called "exact") solutions are computed using the model developed by *Iaquinta* [1995]. This model implements Carlson's quadrature, and we used a total of 144 solid angles in the entire hemisphere corresponding to eight different polar regions. The one-angle problem was solved using a Gauss quadrature scheme with eight directions in each hemisphere. The relative inaccuracies related to the use of a one-angle solution are displayed for a typical canopy (i.e., $r_\ell$ = 0.5 and $t_\ell$ = 0.45) bounded by a dark Lambertian soil ($R_s$ = 0.15) and for four different leaf angle distribution functions. The parameter $\varepsilon$ has been computed for two angles of incoming radiation, namely $\theta_0 = 0°$ and 60°, and for two values of the leaf area index, 1 and 8, respectively, which correspond to two different multiple-scattering regimes.

It can be seen in Figures 2 and 3 that $\varepsilon$ remains less than 1.5% of the multiple-scattering contribution. Accordingly, and considering the gain in computer time, the one-angle solution appears well suited for representing the contribution due to the multiply collided intensity, which barely exceeds 60% of the total scattered intensity emerging from the top of the canopy.

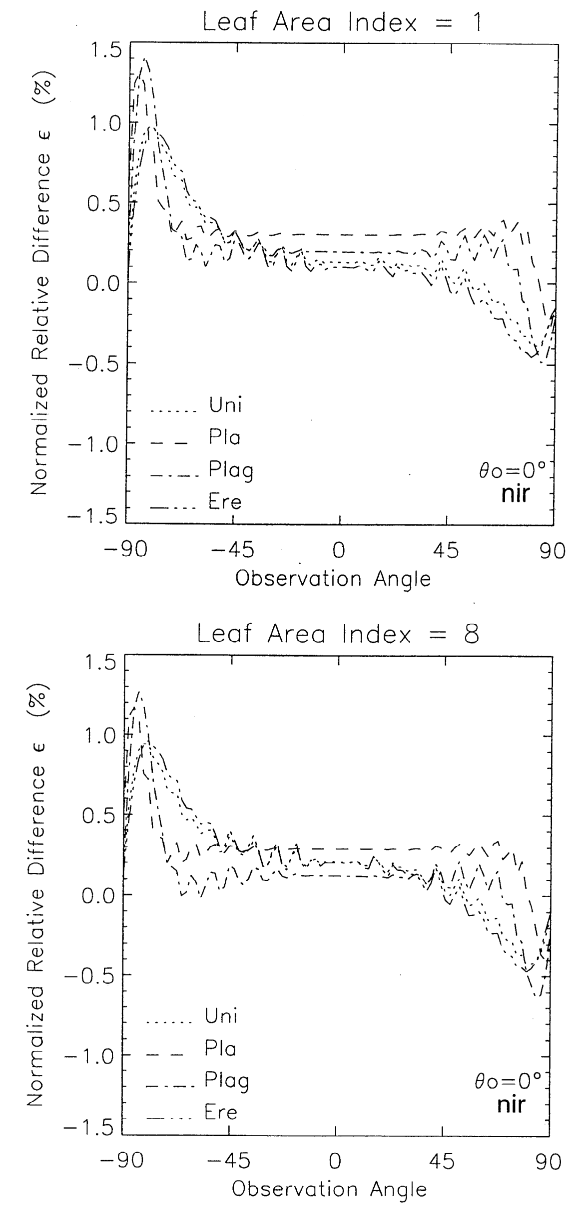


**Figure 2.** Relative differences in the estimate of the multiple-scattering contributions when computed using a "one-angle" scattering phase function instead of the full solution. The plots are drawn in the principal plane for $\theta_0 = 0°$. Abbreviations are as follows: nir, near-infrared; uni, uniform; pla, planophile; plag, plagiophile; and ere, erectophile.

### 3.4. Bidirectional Reflectance Factor of the Canopy

The bidirectional reflectance factor of the canopy (implicitly at level $z_0$) is obtained by summing the three contributions due to the various orders of scattering:

$$\rho(\Omega_0, \Omega) = \rho^0(z_0, \Omega_0, \Omega) + \rho^1(z_0, \Omega_0, \Omega) + \rho^M(z_0, \mu_0, \mu) \tag{37}$$

where the first two terms correspond to the bidirectional reflectance factors due to the zero and first order of scattering by the leaves following a discrete approach, and the third term is the bidirectional reflectance factor due to the multiple scattering, approximated using the assumption of azimuthal symmetry for the scattering phase function of the leaves.

The input variables describing the vegetation system in the semidiscrete model can be separated into a spectral and an architectural category. Any three of the following architectural variables are required: the height of the canopy $H$ (meters), the leaf area index LAI (square meters per square meter), the equivalent diameter $d_\ell$ (meters) of a single leaf of average dimension, and the average number density of leaves $n_v$ ($m^{-3}$). The average orientation of the leaves is specified by a leaf angle distribution function $g(\Omega_\ell)$ which can be evaluated using Bunnik's formulae. These formulae assume an azimuthal symmetry so that $g(\Omega_\ell)$ reduces to $g(\theta_\ell)$. The spectral variables are the leaf reflectance $r_\ell$ and transmittance $t_\ell$, as well as the soil properties; the latter are characterized by a value for the single-scattering albedo, the asymmetry factor, and the hot

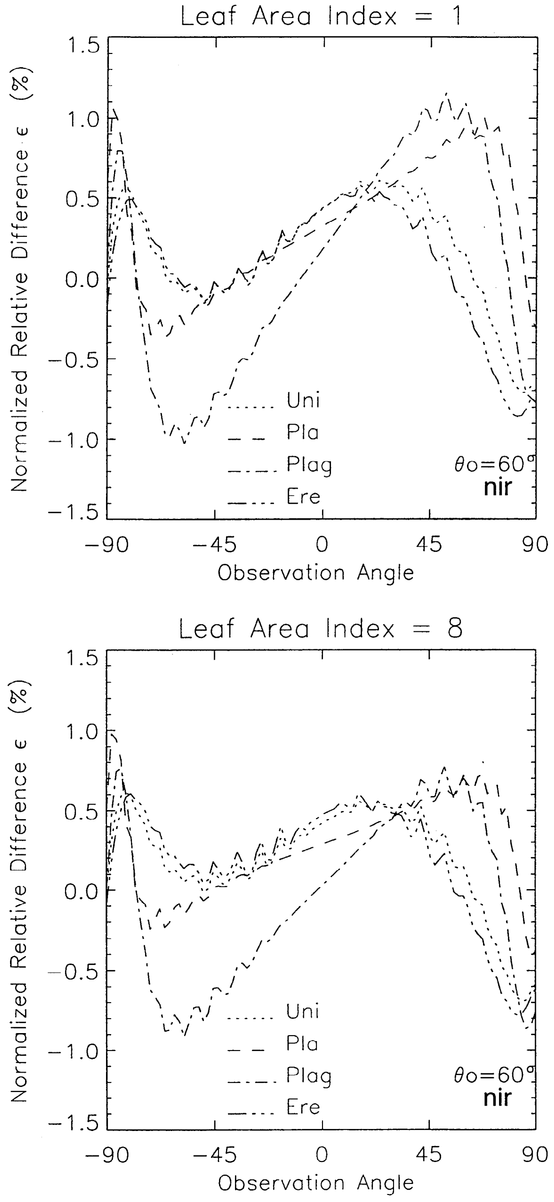


**Figure 3.** Same as Figure 2 but for $\theta_0 = 60°$.

spot parameter, respectively. In the case of a Lambertian soil, the only required variable for the model is the soil albedo, $R_s$.

When the plant canopy system is idealized as a purely turbid medium, the required architectural variables are the leaf area index and the leaf angle distribution function. Therefore our improvement is made at the cost of any two of the three additional variables ($H$, $d_\ell$, or $n_v$) for representing the architecture of the system. The role of these two additional variables can be examined by comparing solutions obtained between a semidiscrete and a turbid approach for the zero and first orders of scattering by the leaves. Results of such a comparison are illustrated in the principal plane (Figure 4) for nine different conditions of canopy architecture displaying small and large leaf sizes as well as dense or sparse canopies and two leaf angle distribution functions. In the case of small-sized leaves, the main differences between the turbid and semidiscrete approach are concentrated around the hot spot angular region. When the size of the leaves increases, the differences concern a much wider angular domain due to the presence of larger spaces free of scatterers or openings in the plant canopy. Note that this comparison is made assuming a typical red wavelength, which implies that the bulk of the signal is carried by the zero and first orders of scattering. Clearly, the situation represented in the lower right panel (5000 leaves of 10 cm $m^{-3}$) is not realistic; the purpose of this panel is only to show graphically how these mathematical functions evolve with the model parameters.

## 4. Evaluation of the Model in a Direct Mode

In order to avoid numerical errors inherent to the use of an inverse method, we choose to first evaluate the model in a direct mode. Ideally, to perform such an experiment requires the availability of a set of bidirectional reflectance factors together with the corresponding set of state variables that enter the model. Accordingly, we examined the model performance under a fully controlled environment as provided by a comprehensive Monte Carlo ray-tracing model. Then, the model predictions are compared to in situ measurements made over a soybean plant canopy for which most of the values of the model variables have been independently assessed or can at least be roughly estimated from ancillary information.

### 4.1. Comparisons With a Monte Carlo Ray-Tracing Model

Since it uses a statistical description of plant canopy with explicit architectural variables, a Monte Carlo ray-tracing model is a suitable tool for evaluating the semidiscrete model. The Raytran model [*Govaerts*, 1996; *Govaerts et al.*, 1995; *Govaerts and Verstraete*, 1995], which can be set up as a "virtual laboratory," computes the bidirectional reflectance factor field emerging from a three-dimensional plant canopy system as a function of the illumination and viewing angles. This model uses a Monte Carlo ray-tracing method, and the architecture of the scene can be accurately described and explicitly represented. In these comparison experiments the scenes are assumed to be horizontally homogeneous, composed of flat oriented disks only and are therefore defined by three geometrical parameters, namely, the height of the canopy $H$, the leaf area index LAI, and the diameter of a single leaf $d_\ell$. This ensures a full compatibility between the two models and permits the assessment of the limitations of the semidiscrete model under idealized situations.

The values of the radiative parameters corresponding to the plant canopies we studied are given in Table 1 for the two wavelengths of major interest in remote sensing, namely, the red and near-infrared. The leaf reflectance and transmittance values are representative of those of a green leaf, and the soil, always assumed to be a Lambertian surface in these cases, is defined by its albedo $R_s$. The values of the architectural variables are chosen to represent canopy conditions in different regions seen in Figure 1. This includes conditions prevailing in the left part of the aforementioned figure with small leaves ($d_\ell = 0.02$ m) and the right part as well with larger leaves ($d_\ell = 0.1$ m) that can bring significant departure from a turbid continuous medium concept. Both dark and bright soils are considered to simulate various effects coming from the lower boundary condition in the case of a low leaf area index value (LAI = 1); we logically limited our investigations to the case of a bright soil for large leaf area index values (LAI = 3 and 8). The specifications of the values for the architectural variables of the various plant canopies are given in Table 2. To limit the

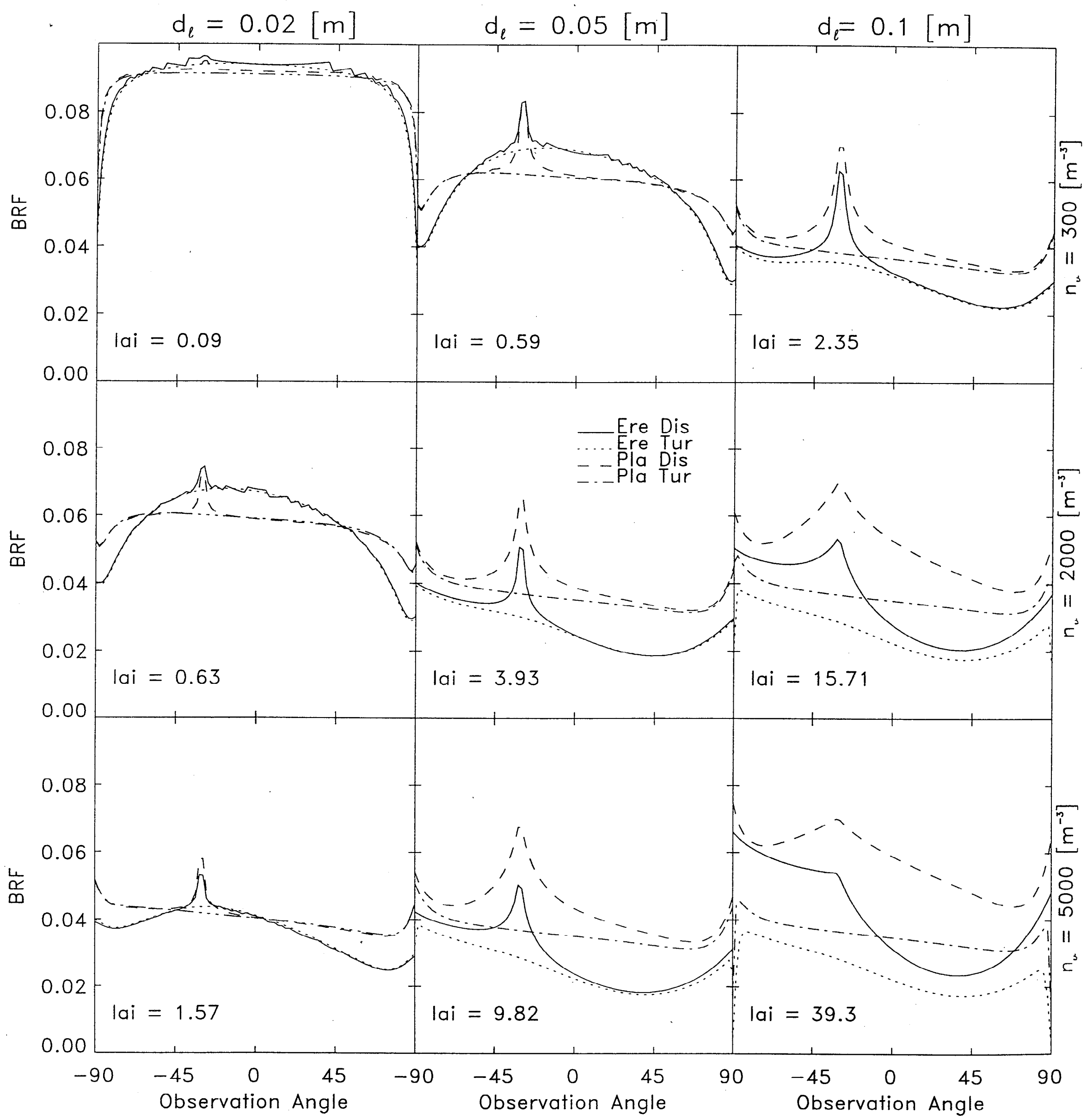


**Figure 4.** Comparison between results obtained following a purely turbid and the semidiscrete modeling. The plots are given in the principal plane for an erectophile (ere, solid and dotted lines) and planophile (pla, dashed and dash-dotted lines) distribution function of the leaf angle. The height of the canopy is 1 m in all cases.

demand on computer resources, Raytran simulations were executed at two solar zenith angles, $\theta_0 = 0°$ and 60°. In the same vein, ensemble averages of radiation transfer over multiple realizations of the scene should have been made, but only one was computed. The evaluation of the semidiscrete model was

**Table 1.** Values of the Spectral Variables Used in the Experiment

| Spectral Property | Red | Near-Infrared |
|---|---|---|
| Leaf reflectance | 0.075 | 0.5 |
| Leaf transmittance | 0.05 | 0.45 |
| Dark soil albedo | 0.1 | 0.15 |
| Bright soil albedo | 0.25 | 0.35 |

**Table 2.** Values of the Architectural Variables Used in the Comparison With Raytran

| LAI | $H$ | $n_\ell$ | Leaf Orientation | Soil | $\theta_0$, deg |
|---|---|---|---|---|---|
| | | | $d_\ell$ = 0.02 | | |
| 1 | 5 | 636 | uni and pla | bright and dark | 0, 30, 60 |
| 3 | | 1908 | pla | bright | 0, 60 |
| 8 | | 5092 | pla | bright | 0, 60 |
| | | | $d_\ell$ = 0.1 | | |
| 1 | 1 | 127 | ere and uni | bright and dark | 0, 60 |
| | 5 | 25 | ere and uni | bright | 0, 60 |
| 3 | 1 | 381 | ere | bright | 60 |
| 8 | 1 | 1018 | ere and uni | bright | 0, 60 |
| | 5 | 203 | ere and uni | bright | 0, 60 |

The leaf orientations are defined as follows: ere, erectophile; pla, planophile; uni, uniform.

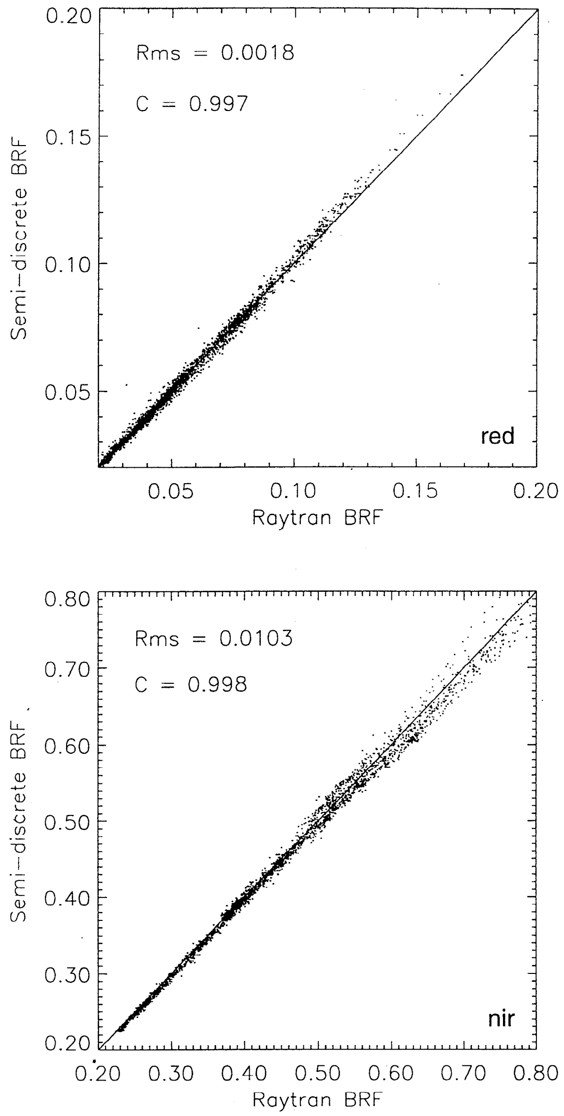


**Figure 5.** Comparison between the results of simulations using the semidiscrete model and Raytran in the (top) red and (bottom) near-infrared domain. The simulations are done on the basis on the values given in Table 1 and Table 2 for the model variables. BRF denotes bidirectional reflectance factor.

carried out in the principal plane, since this is the region with the strongest anisotropy. A solar zenith angle of 30° is also considered in the case of a sparse canopy. The leaf angle distributions are either uniform, erectophile, or planophile.

Figure 5 represents the bidirectional reflectance factors computed by the semidiscrete model as function of the results of Raytran simulations in both spectral domains and for all canopy conditions described above. It can be seen that the differences between the results obtained with the two models are about 5% of the bidirectional reflectance factor values in the red and 3% in the near-infrared on average. A number of reasons may explain the slight discrepancies between the two models, including purely numerical inaccuracies and the neglect of the hot spot effects in the multiple-scattering contribution in the semidiscrete model. The latter should logically lead to an underestimation of the bidirectional reflectance factors generated by this model when compared to the Raytran simulations used as a reference.

Figures 6 and 7 illustrate the bidirectional reflectance factors in the principal plane calculated by the semidiscrete and the Raytran models for three values of the leaf area index, LAI = 1, 3 and 8, in each spectral band. The first case, Figure 6, represents a sparse medium in which the leaves are small ($d_\ell$ = 0.02 m) and mostly horizontal. The canopy height is equal to 5 m, and the source of illumination is located at nadir. The second case (Figure 7) represents a dense canopy with relatively large erectophile leaves (i.e., the diameter of a single leaf is equal to 0.1 m). In this case, the height of the canopy is equal to 1 m, and the illumination zenith angle is equal to 60°. In both cases the simulations are made considering a bright soil condition. The semidiscrete model appears very accurate for representing the bidirectional reflectance factors in the principal plane where most of the variability occurs. This good performance is, as expected, particularly effective in the red simulated wavelength since most of the signal is due to the very first orders of scattering by the soil and the leaves. In the near-infrared domain the inaccuracies discussed above are illustrated in the case of the dense canopy in which case about 40–60% of the signal is due to the multiple-scattering events. Inspection of other cases (not shown here) confirms these results, namely, that the inaccuracies originate mainly from the simplified simulation of the multiple-scattering contribution in the case of dense canopies.

### 4.2. Application to Field Data

The performance of the model for predicting bidirectional reflectance factors in the case of field observations, which supposedly feature more complex properties than those dealt with in the modeling approach, has been examined using the measurements made in the case of a soybean plant canopy [*Ranson et al.*, 1984]. Among the complete available data set, we limited our application to those measurements that were taken under conditions where the soybean canopy offers a quasi-homogeneous architectural structure. Nevertheless, some remaining row effects can be anticipated to alter the ability of the model to fully represent the angular variations observed in the field.

The architectural as well as the optical properties of the soybean canopy measured by Ranson and his colleagues on August 27, 1984, are reported in Table 3, together with some additional values of model variables that we estimated from various ancillary information. For instance, the average size of the leaves has been deduced assuming a uniform density in a limited volume of scatterers (i.e., the canopy contains 348 leaves in a volume of $1.02 \times 1.04$ m$^3$), and we will also consider that the leaf angle distribution function follows a uniform distribution, although in situ measurements show some variability both in the density and orientation of the leaves along the vertical axis.

To solve the radiation transfer problem, it is not sufficient to describe the optical and structural properties of the medium (in this case the vegetation canopy) in which the radiation propagates; it is also necessary to specify the external sources of radiation at the boundary of this medium. In the case of a one-dimensional problem such as treated here, these conditions include the radiation fluxes at the top and bottom of the canopy. Since the radiation may interact multiple times with the overlying atmosphere, the canopy, and the underlying soil, this problem can only be fully addressed with coupled models capable of representing all relevant processes, and in particular the multiple scattering between these media. Considering the extra computational cost involved, a detailed representation of

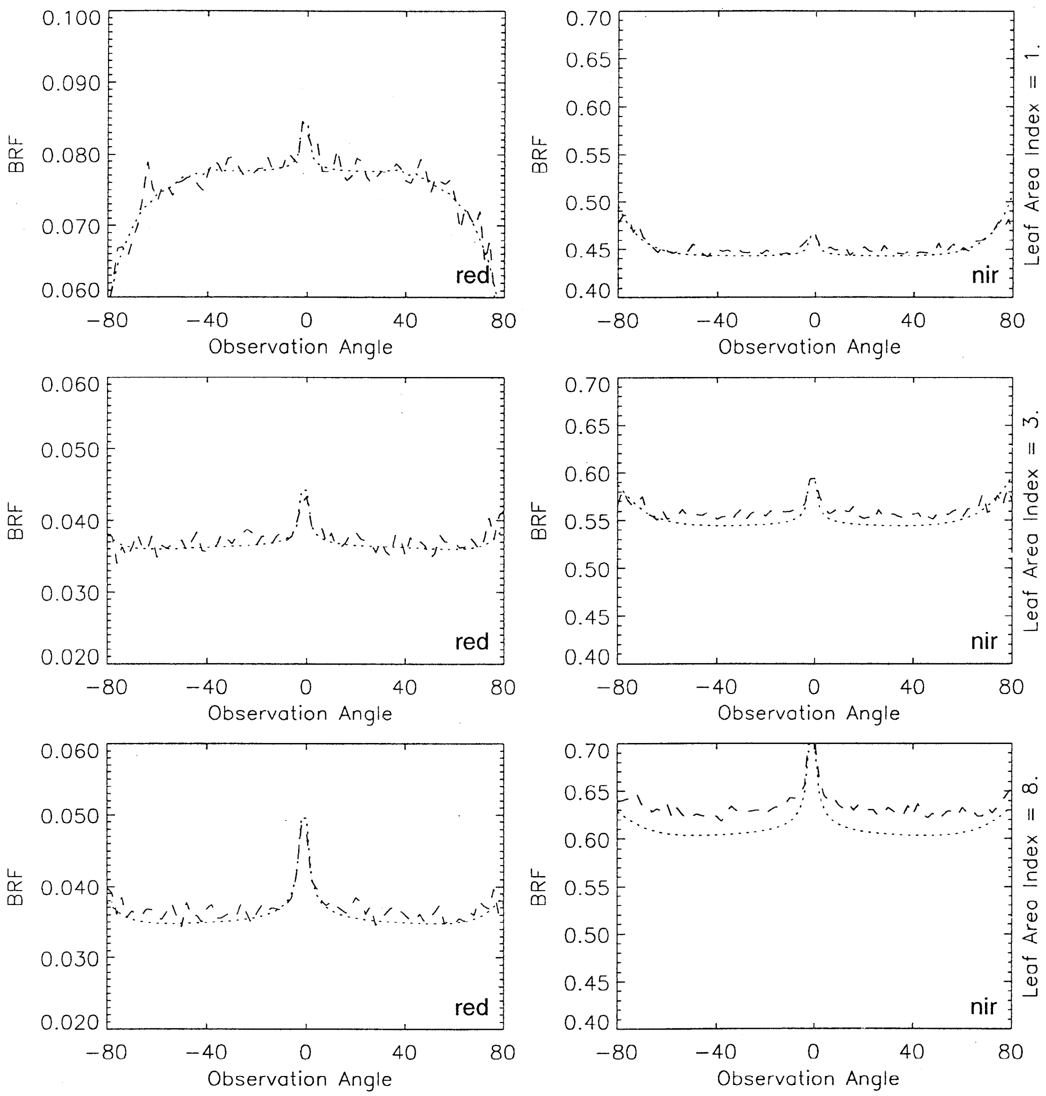


**Figure 6.** Comparison between the bidirectional reflectance factors calculated using the semidiscrete model (dotted lines) and Raytran (dashed lines) in the red domain (left side of the figure) and in the near-infrared domain (right side of the figure) for three leaf area index values. The plots are given in the principal plane with an illumination zenith angle at 0°. They correspond to the case of a bright soil, a planophile leaf angle distribution function, a leaf diameter of 0.02 m, and a canopy height of 5 m.

these conditions as a function of angles should be justified by an accurate knowledge of the radiation field at these boundaries. Since only flux densities instead of intensities were tentatively measured, we adopted a simple approach for the treatment of the boundary conditions. The soil albedo value was estimated from a simple arithmetic average of four directional values measured in situ. This isotropic assumption should not introduce too much error since the leaf area index value of the soybean canopy is large enough to provide a relatively smooth downward field of intensities impinging the top of the underlying soil. The issue of accounting for the atmospheric diffuse radiation in the model simulation has been avoided by adopting an approximate correction scheme for the measured intensities emerging from the canopy as suggested by *Pinty et al.* [1990] after a study from *Tanré et al.* [1983]. This correction approach allows us to remove the contribution of the atmospheric diffuse radiation, providing an assumption on its angular distribution. Accordingly, the measured reflectance factors $R(\Omega_0, \Omega)$ can be related to the bidirectional reflectance factors by

$$R(\Omega_0, \Omega) = \rho(\Omega_0, \Omega) + [\tilde{\rho}(\Omega_0, \Omega) - \rho(\Omega_0, \Omega)]f_d(\Omega_0) \tag{38}$$

where $f_d(\Omega_0)$ is the ratio of diffuse over total irradiance and $\tilde{\rho}(\Omega_0, \Omega)$ is the angular average of the bidirectional reflectance factor $\rho(\Omega_0, \Omega)$ weighted by the diffuse atmospheric flux. *Tanré et al.* [1983] suggested expressing the term $\tilde{\rho}(\Omega_0, \Omega)$ as

$$\tilde{\rho}(\Omega_0, \Omega) \simeq a\rho(\Omega_0, \Omega) + c \tag{39}$$

where the values of the coefficients $a$ and $c$ must be estimated from radiative transfer computations using various realistic surface properties and atmospheric conditions. In the following, we will use the values derived in the case of a moderately

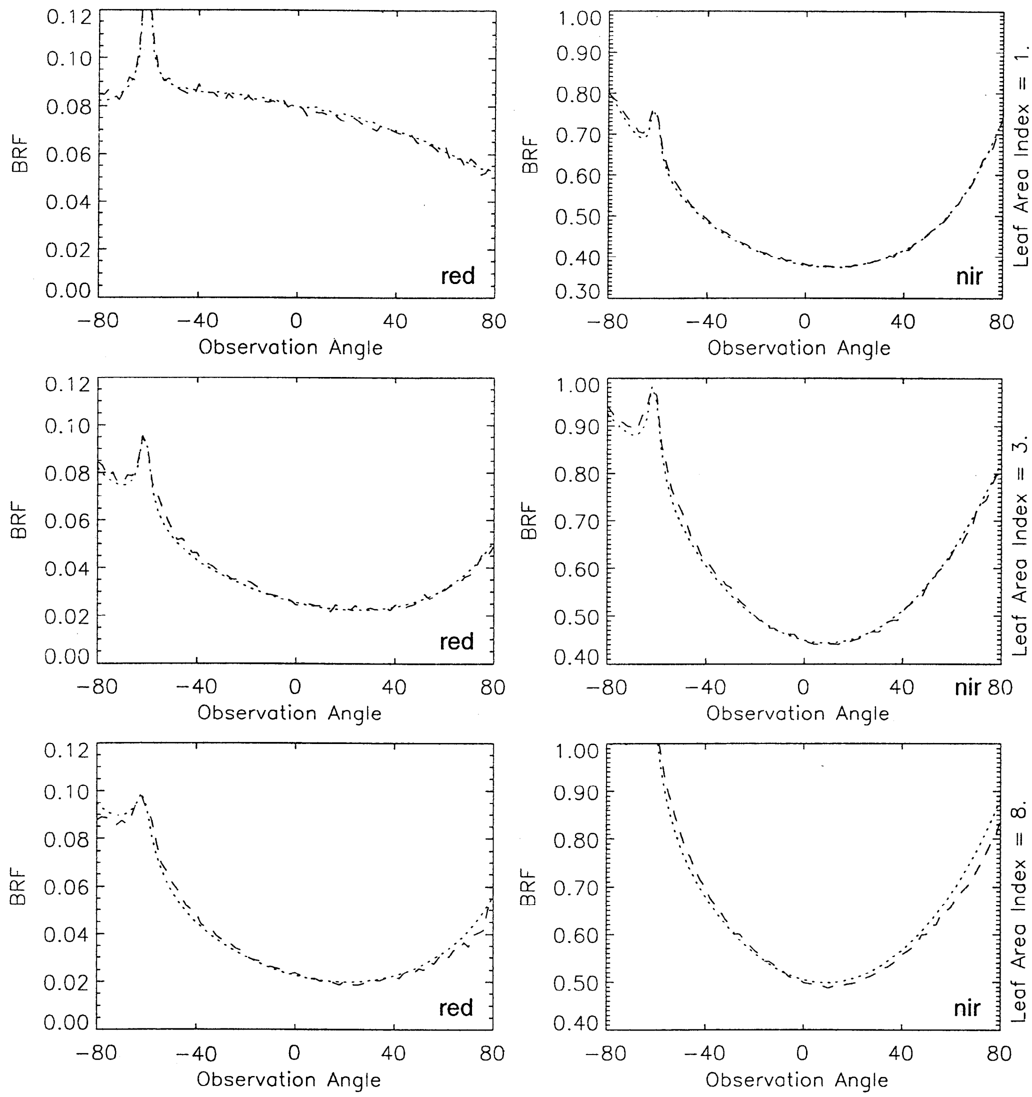


**Figure 7.** Comparison between the bidirectional reflectance factors calculated using the semidiscrete model (dotted lines) and Raytran (dashed lines) in the red domain (left side of the figure) and in the near-infrared domain (right side of the figure) for three leaf area index values. The plots are given in the principal plane with an illumination zenith angle at 60°. They correspond to an erectophile leaf angle distribution function, a leaf diameter of 0.10 m, and a canopy height of 1 m.

hazy continental atmosphere (e.g., a horizontal visibility of 23 km) over a surface which exhibits the same bidirectional reflectance factors as the savannah measured by *Kriebel* [1978]. The fraction of the diffuse flux $f_d(\Omega_0)$ has been assumed to be equal to 0.1 in the red domain when the Sun is located at $\theta_0 = 44°$ and $\phi_0 = 237°$, and the values at other solar positions and other wavelengths have been then deduced from the simplified method for atmospheric correction (SMAC) model [*Rahman and Dedieu*, 1994] to ensure spectral and angular consistency in the variations of the atmospheric diffuse component.

With the information given in Table 3 and the specification of the boundary conditions discussed above, the model can be used in direct mode to simulate the bidirectional reflectance factors in the red and near-infrared wavelengths at the actual angular conditions of observation. Results of the model simulations are compared with the observations in Figure 8 for three solar positions defined by their zenith and azimuth angles, namely, (31°, 196°), (44°, 237°) and (61°, 258°), respectively. The agreement between model simulations and field data is globally good in the near-infrared spectral domain for all three solar zenith angles. The linear correlation coefficient is higher than 0.85 and the root-mean-square of the fit between the two distributions roughly corresponds to 5% of the bidirectional reflectance factors. In the red spectral domain the comparison between the model-simulated values and the observed values shows some significant discrepancies as expressed by low correlation coefficients and root-mean-square values as large as 15–20% of the bidirectional reflectance factors.

These results indicate that the model has some potential to explain both the spectral and the angular variations observed in the bidirectional reflectance factors, and this is particularly true in the near-infrared domain. There are a number of reasons for explaining the discrepancies between the observed and simulated reflectances: (1) the model constitutes an idealization of the actual soybean canopy and (2) some of the values required by the model as input variables may not be known with enough accuracy (i.e., the upper and lower boundary conditions) or have been approximated using available information (i.e., the average size of the leaves and the leaf angle distribution function). We speculate that these errors affect the red spectral band more than the near-infrared, because in this latter case the multiple scattering smears the radiative effects due to the structure of the canopy. All things considered, the soybean canopy used in this study constitutes a very simple case from the architectural point of view, and the model should a priori be capable of explaining the observed angular variations better than shown in Figure 8. Regarding the uncertainties in the values of the input variables, we need to identify what could be the main source of errors, which means the most sensitive state variables which have the largest errors. In this respect, the measured leaf area index takes values between 2.5 and 3.3, corresponding to an optically thick canopy when sensed at the red wavelength; it implies that variations around the average values of the leaf area index and the soil albedo will not drastically change the intensity field emerging at the top of the soybean canopy [*Gobron et al.*, 1997]. Therefore both the leaf area index and the soil albedo values can hardly be the main source of uncertainties to explain the differences between the model simulations and the observations. Consequently, this pair of state variable values cannot be reliably retrieved using an optimization procedure to estimate the best set of values for the model variables to provide a statistically optimal fit to the measurements. Also, the reasonably good agreement between model simulations and observations obtained in the near-infrared domain seems to suggest that the approximation of a uniform distribution for the leaf angle distribution function is acceptable. In fact, additional simulations made by changing this distribution function do not improve the quality of the fits or lead to much worse results than those obtained when assuming a uniform distribution.

To retrieve the most likely values of the model variables that may explain the variability observed in the data, we apply an optimization procedure to minimize the following merit function:

$$\delta^2 = \sum_{i=1}^{m} 4\left[\frac{\rho^i_{\text{data}}(\Omega_0, \Omega) - \rho^i_{\text{model}}(\Omega_0, \Omega)}{\rho^i_{\text{data}}(\Omega_0, \Omega) + \rho^i_{\text{model}}(\Omega_0, \Omega)}\right]^2 \tag{40}$$

where $\rho^i_{\text{data}}(\Omega_0, \Omega)$ and $\rho^i_{\text{model}}(\Omega_0, \Omega)$ correspond to the observed and model bidirectional reflectance factors, respectively. The routine E04JAF from the Numerical Algorithms Group (NAG) library, which implements a quasi-Newton algorithm, was chosen to solve this optimization problem. The minimization procedure was applied using only a small subset of the full data set, so that the rest of the measurements could be used to evaluate the performance of the model. In this case we selected only those data points close to the principal and cross planes, for a solar zenith angle of 44°. Simulated values were, however, generated for all configurations of illumination and observation for which data were acquired, and Figure 8 shows indeed that the bulk of the variability present in the data set can be represented by the model on the basis of this limited data subset.

In order to limit the number of variables to be optimized, we specified the ratio between two of the architectural variables, namely, the height of the canopy ($H$) and the diameter of a single leaf ($d_\ell$). Therefore for a given spectral band and a given leaf angle distribution function, the number of model

**Table 3.** Architectural and Optical Properties of the Soybean Canopy According to *Ranson et al.*'s [1984] Measurements on August 27, 1984

| Property | Value |
|---|---|
| Leaf area index | 2.9 (±0.4) $m^2m^{-2}$ |
| Height of canopy | 1.02 (±0.04) m |
| Average diameter of a leaf* | 0.1065 m |
| Leaf reflectance | 0.073 (red) |
| | 0.464 (near-infrared) |
| Leaf transmittance | 0.064 (red) |
| | 0.518 (near-infrared) |
| Soil albedo† | 0.14 (±0.02) (red) |
| | 0.23 (±0.02) (near-infrared) |

*Value estimated on the basis of other ancillary data.
†Average of measured reflectance values.

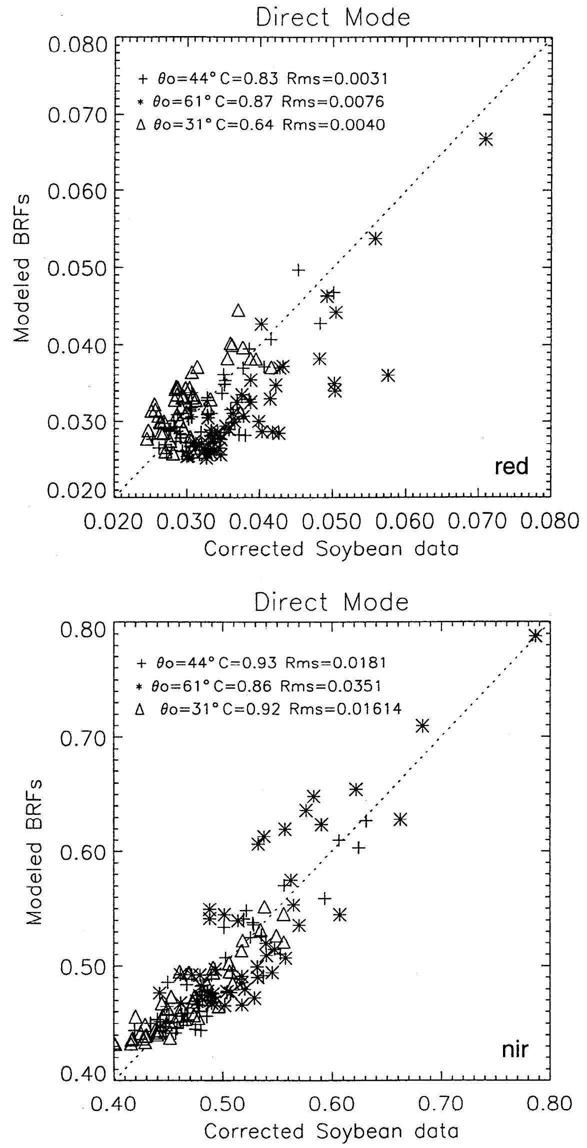


**Figure 8.** Comparison between the bidirectional reflectance factors observed over a soybean canopy and simulated using the semidiscrete model with in situ information for estimating the values of the model variables for the cases of (top) red wavelength and (bottom) near-infrared wavelength.

**Table 4.** Range of Search Allowed in the Inversion Procedure to Retrieve the Optimal Set of Model Variable Values

| Parameters | LAI | $R_s$ | $d_\ell/H$ | $r_\ell$ | $t_\ell$ |
|---|---|---|---|---|---|
| | | *Red Spectral Band* | | | |
| Upper bound | 3.5 | 0.20 | 0.20 | 0.20 | 0.20 |
| Lower bound | 2.0 | 0.05 | 0.05 | 0.010 | 0.010 |
| | | *Near-Infrared Spectral Band* | | | |
| Upper bound | 3.5 | 0.30 | 0.20 | 0.600 | 0.600 |
| Lower bound | 2.0 | 0.15 | 0.05 | 0.400 | 0.400 |

variables has been reduced to five: LAI, $R_s$, $d_\ell/H$, $r_\ell$, and $t_\ell$. The inverse procedure was then tested against a synthetic data set generated by the model itself for the actual conditions of illumination and observation of the soybean canopy. This simple test demonstrated the ability of the procedure to estimate the optimal set of values entering the model given the limited sampling of the emerging intensity field. The search for optimal variable values has been allowed within the bounds which are given in Table 4. It can be seen that a high variability is permitted to the leaf spectral properties, while according to the discussion above on the impact of errors, the leaf area index and the soil albedo are restricted within a domain of values much smaller than their full physical range of natural variability.

The inversion procedure was activated a number of times for various sets of initial guess values, and Table 5 only reports results for which the routine actually stopped on a well-recognized minimum according to its own internal criteria. This set can then be understood as one of the various sets which provide an optimal agreement between the model simulations and the observations according to the merit function we defined. Indeed, there must be a nonunique set of solutions that could provide a good fit to the observations, but within the authorized bounds, the set given in Table 5 is a very probable candidate.

The values given in Table 5 can then be used to simulate the measurements for all conditions of illumination and observation. Figure 9 illustrates the results of this comparison at the red and near-infrared wavelengths, and in both cases the optimized set of the model variable values leads to a much better agreement between the model simulations and the data. The improvement is particularly significant at the red wavelength as indicated by the increase in the values of the correlation coefficient together with root-mean-square values corresponding to roughly 5% of the measured bidirectional reflectance factors. The quality of the fit has only been slightly improved using the optimized set at the near-infrared wavelength. As seen

**Table 5.** Values of the Retrieved Model Variables at the Red and Near-Infrared Wavelengths Using a Subsample of the Original Data Set Taken at $\theta_0 = 44°$ and $\phi_0 = 237°$

| Parameters | LAI | $R_s$ | $d_\ell/H$ | $r_\ell$ | $t_\ell$ |
|---|---|---|---|---|---|
| | | *Red Spectral Band* | | | |
| Estimated values | 2.9 | 0.14 | 0.1044 | 0.073 | 0.064 |
| Retrieved values | 3.39 | 0.05 | 0.1018 | 0.074 | 0.104 |
| | | *Near-Infrared Spectral Band* | | | |
| Estimated values | 2.9 | 0.23 | 0.1044 | 0.454 | 0.518 |
| Retrieved values | 3.09 | 0.29 | 0.1010 | 0.496 | 0.452 |

from Table 5, the optimized values mainly correspond to changes in the leaf reflectance and transmittance values at both wavelengths. Therefore the angular variations in intensity field emerging from the soybean canopy are strongly controlled by the scattering phase function of the leaves, as logically expected in the case of such a canopy. As discussed previously, the retrieved values for the leaf area index and the soil albedo are not very reliable, since changes in their values, which control the lower boundary condition, will not significantly impact the field of intensities at the top of the canopy.

## 5. Conclusion

The radiation transfer model described in this paper represents a compromise between the need to develop tools which are sufficiently accurate and fast for operational uses and the

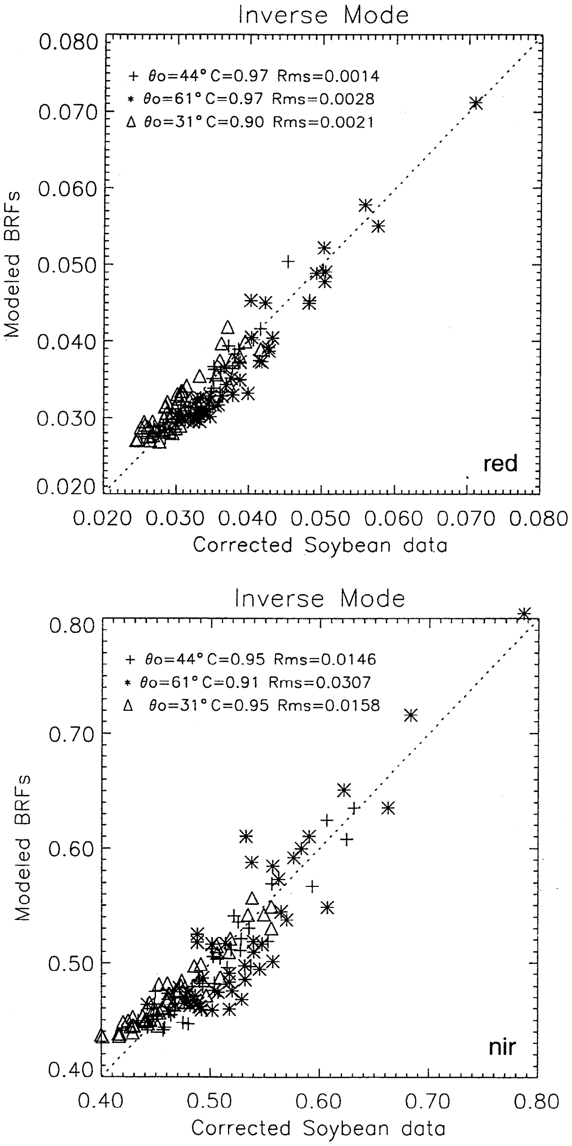


**Figure 9.** Comparison between the bidirectional reflectance factors observed over a soybean canopy and simulated using the semidiscrete model with a set of model variables obtained by inversion of the model against a subset of measurements taken when the Sun was located at 44° for the cases of (top) red wavelength and (bottom) near-infrared wavelength.

desire to represent the relations between the relevant state variables of the radiation transfer problem. This model takes advantage of and merges various previously isolated approaches, namely, the turbid medium concept, the description of the anisotropic effects which result from the finite size of the scatterers, and the representation of a finite plant canopy as a set of layers containing a finite number of leaves of specified size and orientation. In contrast to a Monte Carlo ray-tracing model which offers a very detailed but specific representation of a particular situation, this semidiscrete model proposes a generic solution applicable to the homogeneous canopies that meet its required hypotheses. The model therefore constitutes a good candidate to simulate the reflectance field of typical terrestrial surfaces for the purpose of representing the physical processes at the lower boundary of atmospheric models, as well as for the proper interpretation of high spatial resolution remote sensing data.

The model was shown to be capable of simulating a variety of data sets, both generated with a detailed Monte Carlo ray-tracing model and acquired from field campaigns. The agreement is typically of the order of a few percents of the simulated red and near-infrared reflectances. In contrast to the case of simulated data, where all atmospheric, canopy, and soil properties are known, the representation of reflectances acquired in the field is more complex because not all state variables are measured at the same time as the reflectances. In particular, the variables that define the external sources of radiation at the top and bottom of the canopy are rarely known with enough detail and accuracy. Consequently, the values of some of these variables have been deduced from logical assumptions about the likely conditions of measurement. Despite these hypotheses and uncertainties, the model has been shown to represent the observed variability in reflectance, while maintaining reasonable values of the relevant state variables.

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## References


Bunnik, N. J. J., The multispectral reflectance of shortwave radiation of agricultural crops in relation with their morphological and optical properties, technical report, Mededelingen Landbouwhogeschool, Wageningen, Netherlands, 1978.

Chandrasekhar, S., *Radiative Transfer*, Dover, New York, 1960.

Gobron, N., B. Pinty, M. M. Verstraete, and Y. Govaerts, Theoretical limits to the estimation of the leaf area index on the basis of optical remote sensing data, *IEEE Trans. Geosci. Remote Sens.*, in press, 1997.

Goel, N. S., and D. E. Strebel, Simple beta distribution representation of leaf orientation in vegetation canopies, *Agron. J.*, *76*, 800–803, 1984.

Govaerts, Y., A model of light scattering in three-dimensional plant canopies: A Monte Carlo ray tracing approach, *EUR Rep. 16394 EN*, Space Appl. Inst., Ispra, Italy, 1996.

Govaerts, Y., S. Jacquemoud, M. M. Verstraete, and S. L. Ustin, Modeling plant leaf bidirectional reflectance and transmittance with a 3-D ray tracing approach, in *International Geoscience and Remote Sensing Symposium (IGARSS '95), Quantitative Remote Sensing for Science and Applications, Florence, Italy, 10–14 July 1995*, pp. 1460–1462, Inst. of Electr. and Electron. Eng., New York, 1995.

Govaerts, Y., and M. M. Verstraete, Evaluation of the capability of BRDF models to retrieve structural information on the observed target as described by a three-dimensional ray tracing code, *Proc. SPIE Int. Soc. Opt. Eng.*, *2314*, 9–20, 1995.

Hapke, B., Bidirectional reflectance spectroscopy, 1, Theory, *J. Geophys. Res.*, *86*, 3039–3054, 1981.

Iaquinta, J., Champ de rayonnement émergeant des surfaces terrestres: Modélisation et inversion dans le cas de milieux optiquement finis et couplés avec une couche atmosphérique, Ph.D. thesis, Univ. Blaise Pascal, Aubière, France, 1995.

Iaquinta, J., and B. Pinty, Adaptation of a bidirectional reflectance model including the hot-spot to an optically thin canopy, in *Proceedings of the 6th ISPRS International Symposium on Physical Measurements and Signatures in Remote Sensing, Val d'Isère, France, 17–21 January 1994*, pp. 683–690, Cent. Natl. d'Etud. Spatiales, Toulouse, France, 1994.

Jupp, D. L. B., and A. H. Strahler, A hot spot model for leaf canopies, *Remote Sens. Environ.*, *38*, 193–210, 1991.

Kimes, D. S., Modeling the directional reflectance from complete homogeneous vegetation canopies with various leaf-orientation distributions, *J. Opt. Soc. Am.*, *1*, 725–737, 1984.

Knyazikhin, Y. V., A. L. Marshak, and R. B. Myneni, Interaction of photons in a canopy of finite-dimensional leaves, *Remote Sens. Environ.*, *39*, 61–74, 1992.

Kriebel, K. T., Measured spectral bidirectional reflection properties of four vegetated surfaces, *Appl. Opt.*, *17*, 253–259, 1978.

Kuusk, A., The hot spot effect in plant canopy reflectance, in *Photon-Vegetation Interactions*, edited by R. Myneni and J. Ross, pp. 139–159, Springer-Verlag, 1991.

Liang, S., and A. H. Strahler, Calculation of the angular radiance distribution for a coupled atmosphere and canopy, *IEEE Trans. Geosci. Remote Sens.*, *31*, 491–502, 1993.

Marshak, A. L., The effect of the hot spot on the transport equation in plant canopies, *J. Quant. Spectrosc. Radiat. Transfer*, *42*, 615–630, 1989.

Monsi, M., and T. Saeki, Über den Lichtfaktor in den Pflanzengesellschaften und seine Bedeutung für die Stoffproduktion, *Jpn. J. Botany*, *14*, 22–52, 1953.

Myneni, R., J. Ross, and G. Asrar, A review on the theory of photon transport in leaf canopies, *J. Agric. For. Meteorol.*, *45*, 1–153, 1989.

Myneni, R., A. L. Marshak, and Y. Knyazikhin, Transport theory for a leaf canopy of finite-dimensional scattering centers, *J. Quant. Spectrosc. Radiat. Transfer*, *46*, 259–280, 1991a.

Myneni, R., A. L. Marshak, Y. Knyazikhin, and G. Asrar, Discrete ordinates method for photon transport in leaf canopies, in *Photon-Vegetation Interactions*, edited by R. Myneni and J. Ross, pp. 45–109, Springer-Verlag, New York, 1991b.

Nilson, T., A theoretical analysis of the frequency of gaps in plant stands, *Agric. Meteorol.*, *8*, 25–38, 1971.

Nilson, T., and A. Kuusk, A reflectance model for the homogeneous plant canopy and its inversion, *Remote Sens. Environ.*, *27*, 157–167, 1989.

Norman, J. M., Radiative transfer in vegetation, in *Heat and Mass Transfer in the Biosphere*, edited by D. A. de Vries and N. H. Afgan, pp. 187–206, Scripta, Washington, D. C., 1975.

Pinty, B., and M. M. Verstraete, Modelling the scattering of light by vegetation in optical remote sensing, *J. Atmos. Sci.*, in press, 1997.

Pinty, B., M. M. Verstraete, and R. E. Dickinson, A physical model for predicting bidirectional reflectances over bare soil, *Remote Sens. Environ.*, *27*, 273–288, 1989.

Pinty, B., M. M. Verstraete, and R. E. Dickinson, A physical model of the bidirectional reflectance of vegetation canopies, 2, Inversion and validation, *J. Geophys. Res.*, *95*, 11,767–11,775, 1990.

Rahman, H., and G. Dedieu, SMAC: A simplified method for the atmospheric correction of satellite measurements in the solar spectrum, *Int. J. Remote Sens.*, *15*, 123–143, 1994.

Ranson, K. J., L. L. Biehl, and C. S. T. Daughtry, Soybean canopy reflectance modeling data sets, *Tech. Rep. 071584*, 25 pp., Lab. for Appl. of Remote Sens., Purdue Univ., West Lafayette, Ind., 1984.

Ross, J., *The Radiation Regime and Architecture of Plant Stands*, Dr. W. Junk, Norwell, Mass., 1981.

Ross, J., and A. L. Marshak, Calculation of the canopy bidirectional

reflectance using the Monte-Carlo method, *Remote Sens. Environ.*, *24*, 213–225, 1988.
Shultis, J. K., and R. B. Myneni, Radiative transfer in vegetation canopies with anisotropic scattering, *J. Quant. Spectrosc. Radiat. Transfer*, *39*, 115–129, 1988.
Tanré, D., M. Herman, and P. Y. Deschamps, Influence of the atmosphere on space measurements of directional properties, *Appl. Opt.*, *22*, 733–741, 1983.
Vanderbilt, V. C., Canopy specular reflectance model, *IEEE Trans. Geosci. Remote Sens.*, *GE-23*, 722–730, 1985.
Verstraete, M. M., Radiation transfer in plant canopies: Transmission of direct solar radiation and the role of leaf orientation, *J. Geophys. Res.*, *92*, 10,985–10,995, 1987.
Verstraete, M. M., B. Pinty, and R. E. Dickinson, A physical model of the bidirectional reflectance of vegetation canopies, 1, Theory, *J. Geophys. Res.*, *95*, 11,765–11,775, 1990.

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